

1. Evaluate $\log_{2018} \sin x + \log_{2018} \cos x + \log_{2018} \tan x$ where $x = \pi$.

- A) -1 B) 0 C) 1 D) *undefined* E) NOTA

2. Simplify: $((2^3 + 7^0)^{\frac{3}{2}} + (27^{\frac{1}{3}} - 36^{\frac{1}{2}})^2)^{\frac{1}{2}}$

- A) $2\sqrt{6}$ B) 6 C) 9 D) $\sqrt{6}$ E) NOTA

3. If $\log_a b + \log_b a = 4$, what is the value of $(\log_a b)^2 + (\log_b a)^2$?

- A) 14 B) 15 C) 13 D) 7 E) NOTA

4. Simplify: $\frac{1}{\log_7 2} \div \log_{\frac{1}{3}} \frac{1}{9} + \log_8 7$

- A) $-\frac{1}{6} \log_2 7$ B) $\frac{5}{6} \log_2 7$ C) $\frac{2}{3}$ D) 3 E) NOTA

5. Find the product of all real solutions to $x^{\log_{16} x} = 8$.

- A) 48 B) $4\sqrt{3}$ C) 12 D) $8\sqrt{3}$ E) NOTA

6. Given that $(\log_x 25)(\log_4 49)(\log_{27} x)(\log_{125} 64)(\log_x 81) = 2$, find $\log_7 x$.

- A) $\frac{4}{3}$ B) $\frac{3}{8}$ C) $\frac{8}{3}$ D) $\frac{3}{4}$ E) NOTA

7. The value of $\sqrt{2 + \sqrt{2^2 + \sqrt{2^4 + \sqrt{2^8 + \dots}}}}$, where exponents are the increasing powers of 2, can be expressed as $\frac{\sqrt{a} + \sqrt{b}}{2}$ where a and b are positive integers. Find $|a - b|$.

- A) 6 B) 8 C) 10 D) 12 E) NOTA

8. Solve for x : $\ln(1 + \log_2(3 + \log_4(5 + x))) = 0$

- A) $-\frac{79}{16}$ B) -1 C) $-\frac{319}{64}$ D) $-\frac{19}{4}$ E) NOTA

9. The roots of $9x^3 - 28x^2 + 81x - 3 = 0$ are r , s , and t . Find $\log_3 r + \log_3 s + \log_3 t$.

- A) 3 B) -2 C) 1 D) -1 E) NOTA

10. If $x = \sqrt[5]{-41}$, then how many of the following are false?

- I. $x > -2$ II. $x^3 \leq -8$ III. $\sqrt{-x} > 1$ IV. $x^2 < 4$ V. $x^4 \geq 16$
- A) 1 B) 2 C) 3 D) 4 E) NOTA

11. Let the roots of the function $f(x) = x^{20} - 7x^3 + 1$ be $r_1, r_2, r_3 \dots r_{20}$. If

$$\frac{a}{b} = \frac{1}{1+r_1^2} + \frac{1}{1+r_2^2} + \frac{1}{1+r_3^2} + \dots + \frac{1}{1+r_{20}^2}$$

where a and b are relatively prime positive integers. Find $a - b$.

- A) 134 B) 196 C) 81 D) 45 E) NOTA

12. Simplify $(\log_{2a} 4^x)(1 + \log_2 a)$.

- A) x^2 B) 2^x C) x D) $2x$ E) NOTA

13. Solve for x : $3^{x-4} = 4^{x-3}$

- A) $\frac{\ln 8 - \ln 9}{\ln 3 - \ln 2}$ B) $\frac{\ln 81 - \ln 64}{\ln 4 - \ln 3}$ C) $\frac{\ln 64 - \ln 81}{\ln 4 - \ln 3}$ D) $\frac{\ln 9 - \ln 8}{\ln 3 - \ln 2}$ E) NOTA

14. Let $F(x) = 2^x + 7$. The graph of a function $G(x)$ is produced by expanding the graph of $F(x)$ vertically by a factor of 8 against the x -axis and then shifting the resulting graph 8 units up. Which of the following is an equation of $G(x)$?

- A) $G(x) = 2^{3x} + 15$ C) $G(x) = 2^{3x} + 64$ E) NOTA
 B) $G(x) = 2^{x+3} + 15$ D) $G(x) = 2^{x+3} + 64$

15. Given $\log_{n^2} m + \log_{m^2} n = 1$, solve for n in terms of m if $n \neq 1, m \neq 1$, and m is positive.

- A) \sqrt{m} B) m^2 C) m D) $\frac{1}{m}$ E) NOTA

16. Given $a \log_{1440} 5 + b \log_{1440} 2 + c \log_{1440} 3 = d$, where a, b, c , and d are relatively prime positive integers, find the value of $ab + cd$.

- A) 11 B) 9 C) 8 D) 7 E) NOTA

17. Let $a + \sqrt{b} = \sqrt{6 + (1 + \sqrt{3 + (1 + \sqrt{3 + \sqrt{8}})^2})^2}$ where a and b are positive integers.
Find $a + b$.

- A) 8 B) 9 C) 13 D) 10 E) NOTA

18. Given $\log_x y + \log_y x = \frac{10}{3}$ and $xy = 400$ where $x > y > 0$, find $x - y$.

- A) $8\sqrt{2}$ B) $38\sqrt{5}$ C) $35\sqrt{2}$ D) $16\sqrt{5}$ E) NOTA

19. Given $\log_4(\log_{64} x) = \log_{64}(\log_4 x)$, find the value of $(\log_2 x)^2$.

- A) 48 B) 72 C) 192 D) 108 E) NOTA

20. How many digits are in the decimal form representation of 18^{50} ?

- A) 61 B) 62 C) 63 D) 64 E) NOTA

21. Find the sum of the digits of the greatest integer less than $(3 + \sqrt{7})^6$.

- A) 19 B) 18 C) 21 D) 20 E) NOTA

22. Evaluate:

$$\frac{\begin{vmatrix} 0 & 0 & e^1 \\ 0 & e^3 & e^2 \\ e^1 & e^2 & e^3 \end{vmatrix}}{\begin{vmatrix} e^3 & 0 & 0 \\ e^2 & e^3 & 0 \\ e^1 & e^2 & e^3 \end{vmatrix}} + \frac{\begin{vmatrix} e^3 & e^2 & e^1 \\ e^2 & e^3 & 0 \\ e^1 & 0 & 0 \end{vmatrix}}{\begin{vmatrix} e^3 & e^2 & e^1 \\ 0 & e^3 & e^2 \\ 0 & 0 & e^3 \end{vmatrix}}$$

- A) 0 B) $\frac{2}{e}$ C) $-2e^{-4}$ D) $2e^{-4}$ E) NOTA

23. Find the units digit of 3^{57^9}

- A) 1 B) 3 C) 7 D) 9 E) NOTA

24. Find the units digit of $\sum_{n=1}^{2017} n^{2017} + \sum_{n=1}^{2018} n^{2018}$

- A) 4 B) 2 C) 9 D) 6 E) NOTA

25. Let $F(n) = 1^n + 2^n + 3^n + 4^n$. For how many integer values of n , where $1 \leq n \leq 2018$, is $F(n)$ a multiple of 5?

- A) 1514 B) 1513 C) 1009 D) 404 E) NOTA

26. Rick the mad scientist is bored and creates a new zinc isotope called Zn-R. He conducts an experiment beginning with 2.8 kg of Zn-R and 6.4 kg of Zn-71. After 12 minutes, he finds that there are exactly 25 more grams of Zn-71 than of Zn-R. Given that the half-life of Zn-71 is 144 seconds, find the half-life of Zn-R in seconds.

- A) 90 B) 120 C) 180 D) 240 E) NOTA

27. Find the sum of all real solutions:

$$\log_2(-x^2 + 7x - 10) + \sqrt{\cos(\pi\sqrt{x^2 + 7}) - 1} = 1$$

- A) 7 B) 9 C) 11 D) 13 E) NOTA

28. Given $f(x) = 2 - \log_3(1 - 4x)$, find the shortest distance between the vertical asymptote and x -intercept of $f(x)$.

- A) $\frac{17}{4}$ B) $\frac{7}{4}$ C) $\frac{15}{4}$ D) $\frac{9}{4}$ E) NOTA

29. Evaluate: $\sqrt[3]{9 \cdot \sqrt[5]{81 \cdot \sqrt[7]{729 \cdot \sqrt[9]{6561 \dots}}}}$, where each radicand is the next power of 9, and the indices of the radicals are the positive odd integers in increasing order.

- A) $3^{\frac{4}{3}}$ B) $3^{\frac{2}{3}}$ C) 3 D) $3^{\frac{3}{2}}$ E) NOTA

30. Let $\log_2(-2 + \sum_{x=1}^{100} x2^x) = a + \log_c b$ where a, b , and c are integers and $a > b > c > 0$. Find $a + b + c$.

- A) 202 B) 201 C) 200 D) 199 E) NOTA